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# The oscillation structure of the Hall current in the presence of a contact surface

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**Abstract.** The behaviour of quasi-Landau levels and the Hall current in the case of two magnetic media divided by a barrier at the contact surface is investigated. It is shown that the resonant underbarrier interaction between the quasi-Landau states located in different magnetic media results in avoided crossings of the energy levels and in oscillation of the related Hall current.

In previous papers [1, 2] the energy-level distribution for a particle in a piecewise linear oscillator potential with a $\delta$-function-type barrier has been studied. This situation appears for instance if the piecewise magnetic homogeneous film is placed in orthogonal uniform magnetic $\boldsymbol{H}$ and electric $\boldsymbol{E}$ fields. Here we consider the dependence of the quasi-Landau energy levels and Hall current on external fields as well as their space behaviour.

As shown in figure 1 the medium where the particle is located consists of two films semi-infinite along the $y$ axis with magnetic permeabilities $\mu_1$ and $\mu_2$ respectively, which are divided by a $\delta$-function-type barrier on the contact surface at $y = 0$. The Schrödinger equation of the problem can be presented in the form

$$\left[\frac{1}{2m}(-i\hbar\nabla - e\boldsymbol{A}_i)^2 + U(z) + \alpha\delta(y) - e\mathcal{E}\cdot y\right]\Psi(x, y, z) = E\Psi(x, y, z) \tag{1}$$

where $U(z)$ is the potential well along the $z$ axis, $\alpha$ is the strength of the $\delta$ barrier, $\boldsymbol{E}(0, \mathcal{E}, 0)$ is the uniform electric field, $\boldsymbol{A}_i$ is the vector potential in the first ($i = 1$, $y > 0$) and second ($i = 2$, $y < 0$) medium, and $e$ and $m$ are the charge and the mass of the particle respectively; the relationship between the vector potential and the magnetic induction is

$$\boldsymbol{B}_i = \operatorname{rot}\boldsymbol{A}_i. \tag{2}$$

Magnetic induction $\boldsymbol{B}_i$ is connected with the strength of the uniform magnetic field $\boldsymbol{H}$ by the relation $\boldsymbol{B}_i = \mu_i\boldsymbol{H}$. The presence of $\delta$-function-type barrier in the Schrödinger equation (1) can be replaced by the boundary condition

$$\Psi(x, +0, z) = \Psi(x, -0, z)$$
$$[\partial\Psi(x, y, z)/\partial y]|_{y=+0} - [\partial\Psi(x, y, z)/\partial y]|_{y=-0} = (2m\alpha/\hbar^2)\Psi(x, 0, z). \tag{3}$$

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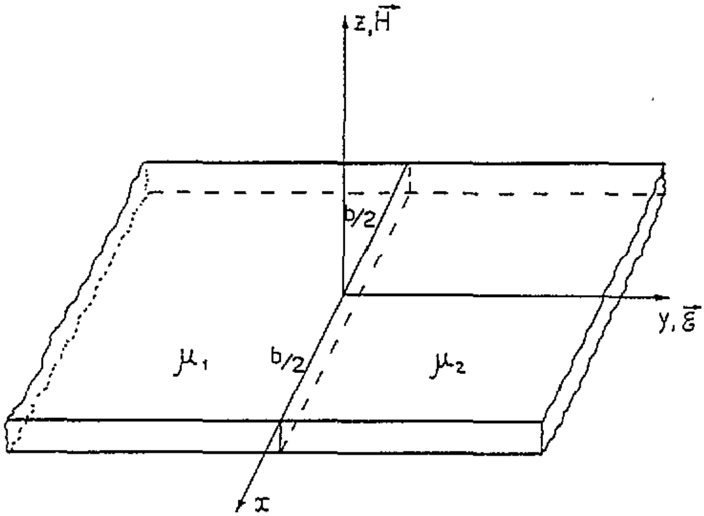


**Figure 1.** The medium in which the charged particle moves: $\mu_1$ and $\mu_2$ are the magnetic permeabilities and $\vec{H}_0$ is the constant magnetic field.

If the vector potential is chosen in the form $A_i = (-\mathcal{B}_i y, 0, 0)$ then the Schrödinger equation (1) can be written as ($y \neq 0$)

$$[-(\hbar^2/2m)\nabla^2 - (\mathrm{i}e\hbar/m)y\mathcal{B}_i\partial/\partial x + (e^2\mathcal{B}_i^2/2m)y^2 - e\mathcal{E}y + U(z)]\Psi(x, y, z) = E\Psi(x, y, z). \tag{4}$$

The variables in (3) and (4) are separated, the solution $\Psi(x, y, z)$ being of the form

$$\Psi(x, y, z) = \mathrm{e}^{(\mathrm{i}/\hbar)xP_x} f_k(z)\varphi_n(y) \qquad (i = 1, 2) \tag{5}$$

where $n$ and $k$ are the quantum numbers of $y$ and $z$ one-dimensional problems respectively and $P_x$ is the generalized momentum along the $x$ axis. If the film has macroscopic size $b$ along the $x$ axis, for periodic boundary conditions the generalized momentum

$$P_x = 2\pi\hbar l/b \qquad (l = 0, \pm 1, \pm 2, \ldots) \tag{6}$$

takes on the quasi-continuous series of values. We also assume that $U(z)$ contains only the ground state ($k = 0$) to separate its spectrum from the spectrum under consideration.

Thus, the non-trivial part of the problem is reduced to the one-dimensional Schrödinger equation for $\varphi_n(y)$

$$[-(\hbar^2/2m)\mathrm{d}^2/\mathrm{d}y^2 + (m\omega_i^2/2)y^2 - e\mathcal{E}y + \omega_i P_x y + P_x^2/2m]\varphi_n(y) = E_n\varphi_n(y) \tag{7}$$

where

$$\omega_i = e\mathcal{B}_i/m \tag{8}$$

$$E_n = E - E_0 \tag{9}$$

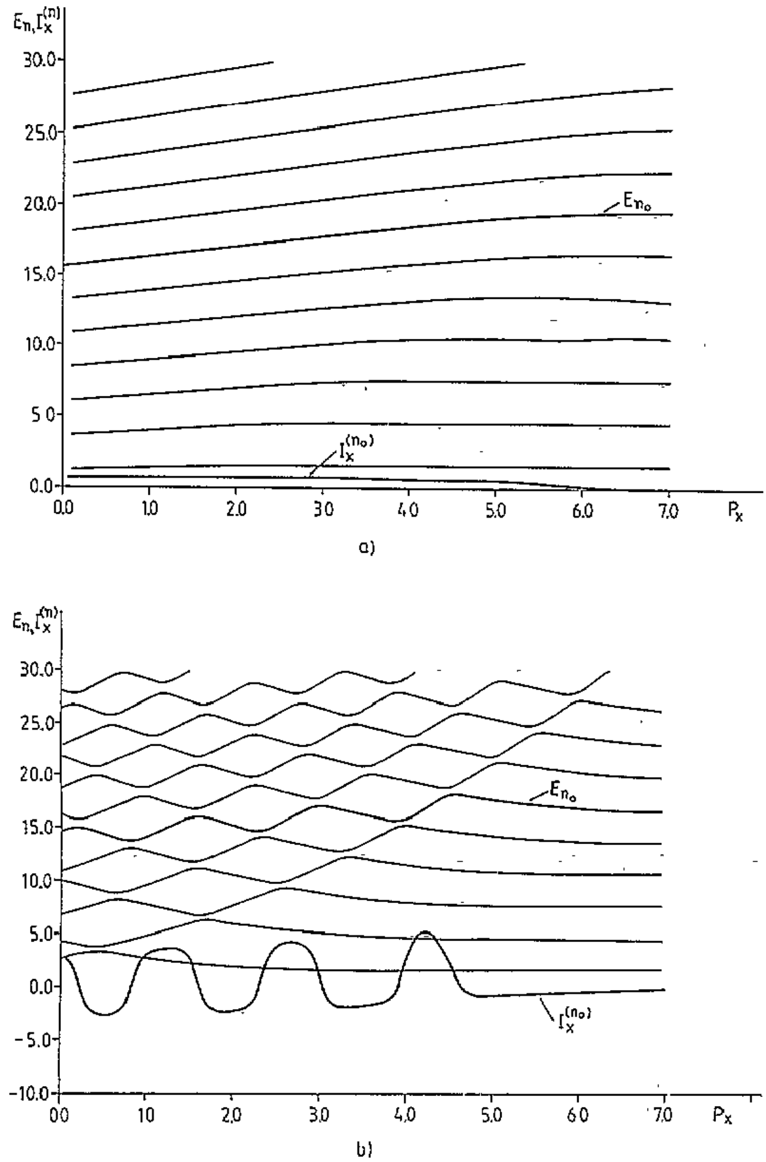


**Figure 2.** The energy curves $E_n$ as functions of the generalized momentum $P_x$ at $\mathcal{E} = 0$ and $\omega_1/\omega_2 =, \frac{2}{3}$: (*a*) $\alpha = 0$; (*b*) $\alpha = 10$. All quantities are presented in atomic units: $m = \hbar = e = 1$. Bold lines indicate individual energy levels and the related Hall current $I_x^{(n)}$.

and $E_0$ is the energy of the ground state in the short-range potential $U(z)$. The general solutions of (7) in each region can be expressed in terms of the function of a parabolic cylinder. The wave function of the problem is obtained by matching this solution at $y = 0$ according to the boundary conditions (3). As a result we obtain a dispersion equation [2]

$$\sqrt{\omega_1} D_{p_1}(-q_1)\{\tfrac{1}{2}q_2 D_{p_2}(q_2) - D_{p_2+1}(q_2)\} - \sqrt{\omega_2} D_{p_2}(q_2)\{\tfrac{1}{2}q_1 D_{p_1}(-q_1) + D_{p_1+1}(-q_1)\}$$
$$= \alpha(2m/\hbar^3)^{1/2} D_{p_1}(-q_1) D_{p_2}(q_2) \qquad (10)$$

where $q_i = (2/m\omega_i^3\hbar)^{1/2}(e\mathcal{E} - \omega_i P_x)$, $p_i = (2E_n - 1)/2\hbar\omega_i$ and $D_p(q)$ is the function of

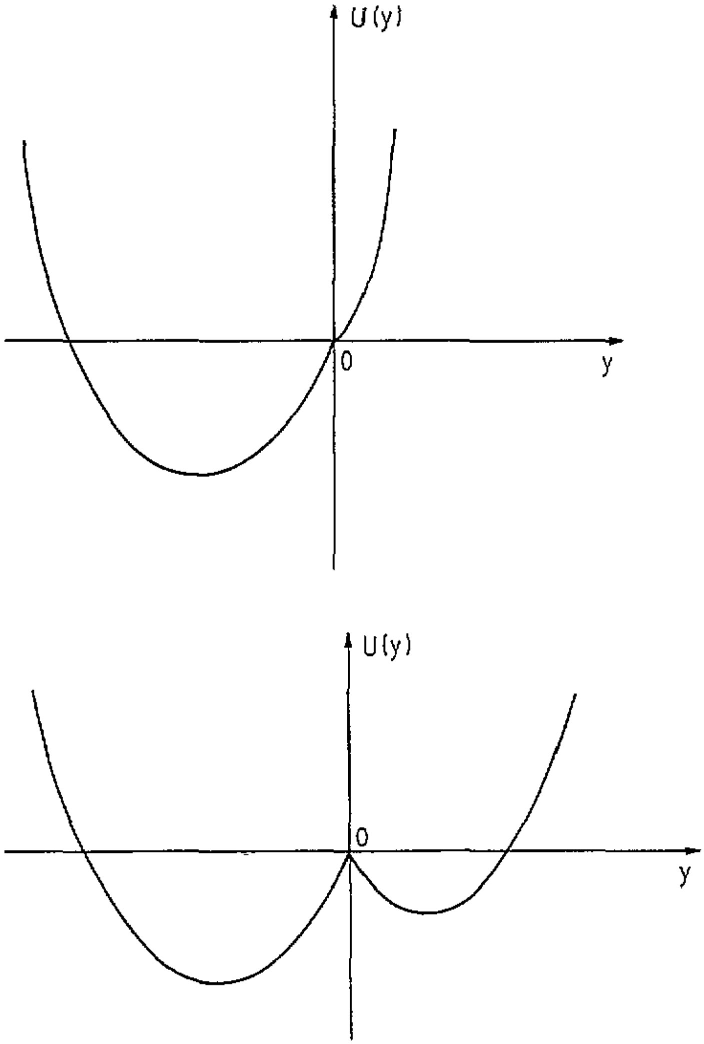


**Figure 3.** Two typical diagrams: the potential along the $y$ axis has the shape of two parabolic wells, divided by a $\delta$ barrier at $y = 0$.

a parabolic cylinder as defined in [3]. (10) determines the energy spectrum of a charged particle in orthogonal electric and magnetic fields in the presence of a barrier at the contact surface. The energy values depend on the following parameters: $E_n = E_n(B_1/B_2, \alpha, P_x, \mathcal{E})$. (10) has been employed to analyse the level spacing distribution for various values of both the $\delta$-barrier factor $\alpha$ and of the ratio $\omega_1/\omega_2 = B_1/B_2$, in the case $\mathcal{E} = 0$, $P_x = 0$ [1] as well as for arbitrary $\mathcal{E}$, $P_x$ [2].

In quite a general case, one can assume that the media in the two regions have different dielectric constants. In such a case, obviously, a piecewise electric field also exists. So, one should write $\mathcal{E}_i$ $(i = 1, 2)$ instead of $\mathcal{E}$. However, it may be easily shown that the net effect of the existence of this field is reflected only through the changes of the position of the parabolic wells in (7).

The current density in an electromagnetic field is given by

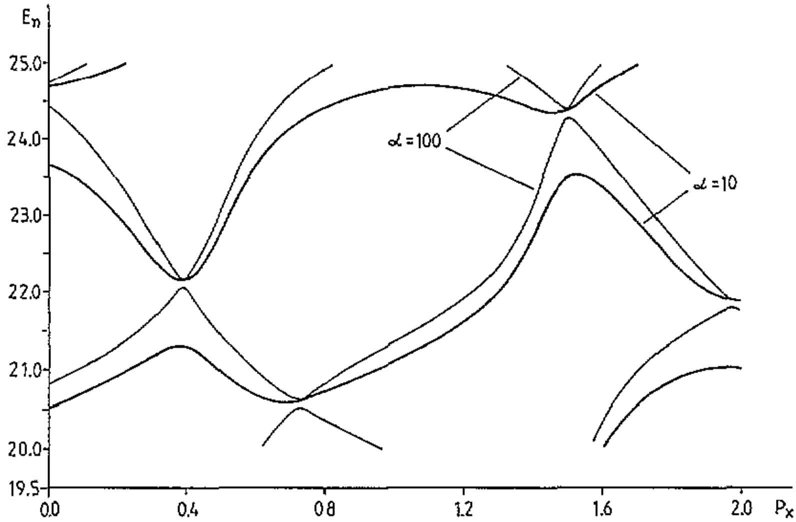


**Figure 4.** The avoided crossing of energy levels at $\omega_1/\omega_2 = \frac{2}{3}$. $\mathcal{E} = 1$ and two values of the barrier factor $\alpha = 10$ and 100.

$$j_i = (\mathrm{i}e\hbar/2m)(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi) - (e^2/m)\boldsymbol{A}_i\Psi\Psi^*$$

where $\Psi^*$ is the complex conjugate wave function. Since $A_i = (-\mathcal{B}_i y, 0, 0)$, the current has only an $x$ component and the related density is

$$j_{ix}^{(n)} = [(e/m)P_x + e\omega_i y]f_0^2(z)\varphi_n^2(y). \tag{11}$$

The partial current along the $x$ axis is obtained by integrating (11) in the orthogonal plane over $y$ and $z$

$$I_x^{(n)} = \int_{-\infty}^{\infty}\int_0^d j_{ix}^{(n)}\,\mathrm{d}y\,\mathrm{d}z = \frac{e}{m}P_x + e\int_{-\infty}^{\infty}\omega_i y\varphi_n^2(y)\,\mathrm{d}y \tag{12}$$

where $d$ is the thickness of the film.

Using the Gell-Mann–Feynman theorem

$$\langle\Psi_n|\partial H/\partial\lambda|\Psi_n\rangle = \partial E_n/\partial\lambda$$

for the Schrödinger equation (1) with $P_x$ as the parameter $\lambda$, expression (12) can be presented in the final form

$$I_x^{(n)} = e(\partial/\partial P_x)E_n(\mathcal{E}, P_x). \tag{13}$$

Figure 2 shows the results of numerical calculation of energy levels and Hall current as functions of generalized momentum $P_x$, which is connected to the position of the parabolic wells in (7) by the relation

$$y_i = (e\mathcal{E} - \omega_i P_x)/m\omega_i^2. \tag{14}$$

Since the wave function is located in the vicinity of the bottom of the potential well, figure 2 gives us in fact the space distribution of partial currents $I_x^{(n)}$ with respect to quantum numbers

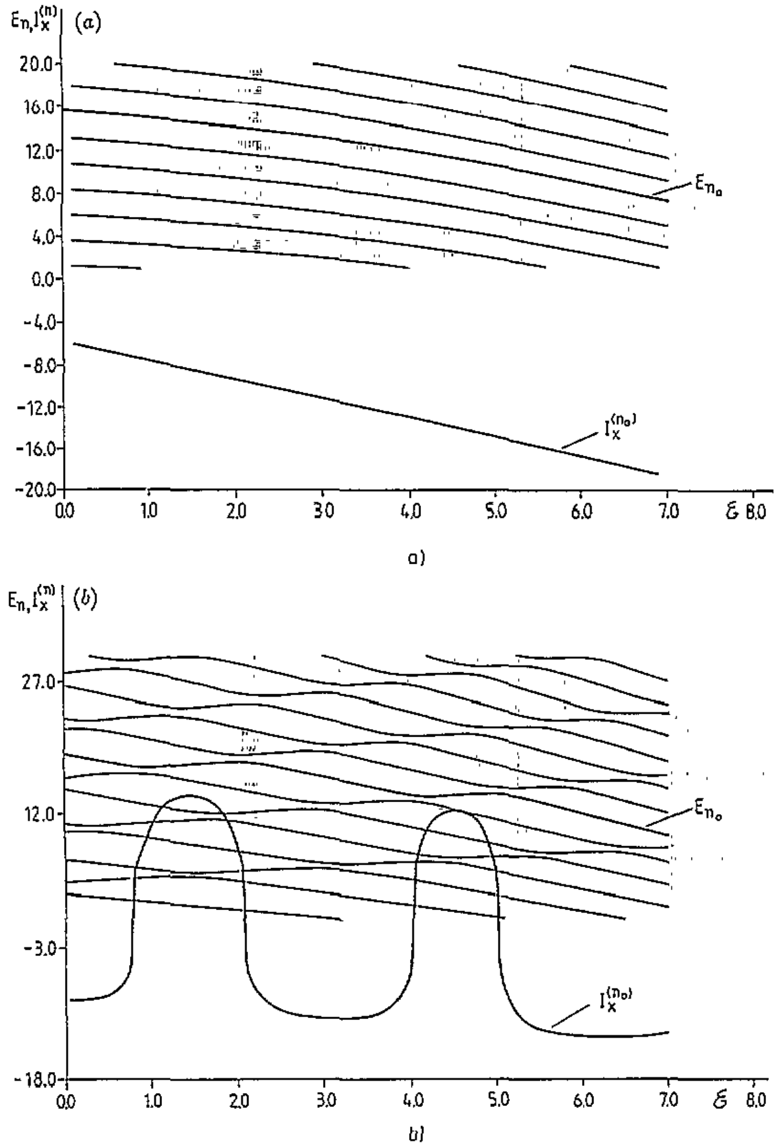


**Figure 5.** The energy curves $E_n$ as a function of the electric field $\mathcal{E}$ at generalized momentum $P_x = 0$ and $\omega_1/\omega_2 = \frac{2}{3}$: (*a*) $\alpha = 0$; (*b*) $\alpha = 10$. All quantities are presented in atomic units: $m = \hbar = e = 1$. Bold lines indicate individual energy levels and the related Hall current $I_x^{(n)}$.

$l$ (see (6)). As one can see from figure 2(*a*) in the absence of a barrier the curves $E_n(P_x)$ are monotonic functions and the related Hall current never changes its direction (the gradient drift along the contact surface exists even if the electric field is absent and the notation 'Hall current' will be used in this case also). At moderate value of $\alpha$ (figure 2(*b*)) the pattern of the energy curves exhibits the avoided-crossings structure. The explanation of this phenomenon is as follows. The effective potential along the $y$ axis has the shape of two potential wells divided by a $\delta$ barrier at the contact surface (figure 3). If one neglects underbarrier interaction the two energy levels of the states located in different potential wells may cross in the course of changing an external parameter (for instance $P_x$). However according to the Neumann–Wigner theorem [4] the exact crossing of the energy levels is prohibited in one-dimensional problems (or in many-dimensional problems for states having the same symmetry). More accurate examination reveals that if two levels approach each other they enter resonance and the underbarrier interaction plays a dominant role. The resonant underbarrier interaction prevents merging of the levels and results in avoided crossing. It should be pointed out the stronger the barrier, the smaller the gap between the levels (see figure 4). In this region the wave functions of given states alternate their location in the potential wells when the value of the parameter (in our case $P_x$) passes the avoided crossing. In other words the barrier becomes absolutely transparent for both states at this point. This phenomenon is well known in the adiabatic approach of atomic-collision theory where avoided crossings of adiabatic potential curves cause intensive inelastic transitions between electron states during the collision (the so-called Landau–Zener transition [5]). In our case owing to relation (13) the avoided crossings of energy levels lead straightforwardly to an oscillatory structure of the current.

The energy curves $E_n(\mathcal{E})$ have similar behaviour. Figure 5 shows the results of numerical calculation of energy levels and Hall current as a function of external electric field. One can see the same avoided-crossing structure of the energy levels as well as oscillation in the related current. In principle this means the direction and location of the partial current can be governed by changing the strength of the electric field. It should be emphasized that the investigated phenomena take place in the vicinity of the contact surface. If $|y_i| \to \infty$ the parabolic effective potential screens the contact surface and the energy levels transform into an equidistant Landau spectrum (see figures 2 and 5).

Obviously the effects considered here are not only related to the $\delta$-barrier model but should also exist in a more realistic description of the barrier at the contact surface.

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